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MTH 205, Differential Equations , Exam One

Ayman Badawi

QUESTION 1. (28 points)

(i) $\ell\{(x+2)^2\} = \ell\{x^2 + 4x + 4\}$

$= \frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s}$

(ii) $\ell\{(x-2)(e^x+1)\} = \ell\{xe^x + x - 2e^x - 2\}$

$= \ell\{xe^x\} + \ell\{x\} - 2\ell\{e^x\} - 2\ell\{1\}$

$= \frac{1}{(s-1)^2} + \frac{1}{s^2} - \frac{2}{s-1} - \frac{2}{s}$

$\ell\{3x^2\} = \frac{2 \cdot 3}{s^3} = \frac{6}{s^3}$

(iii) $\ell\{\int_0^x 3r^2 e^{-2x-3r}\ dr\}$

$= \ell\{\int_0^x 3r^2 e^{-2(x-\frac{3}{2}r)}\ dr\}$ 5/11

7 $= \ell\{\int_0^x 3r^2 e^{-2(x-r)} \cdot e^{-r}\}$

$= \ell\{e^{-x}3x^2 * e^{-2x}\} = \frac{6}{(s+1)^3} \cdot \frac{1}{s+2}$

(iv) $\ell^{-1}\{\frac{s+6}{s^2+4s+20}\}$

$\ell^{-1}\left\{\frac{s+6 (+2-2)}{(s+2)^2+16}\right\} = \ell^{-1}\left\{\frac{(s+2)+4}{(s+2)^2+16}\right\} = e^{-2x}(\cos 4x + \sin 4x)$

$* \ell^{-1}\left\{\frac{s}{s^2+16}\right\} + \ell^{-1}\left\{\frac{4}{s^2+16}\right\} = \cos 4x + \sin 4x$

(v) $\ell^{-1}\{\frac{e^{-2s}}{s^2+6s-7}\}$

$\ell^{-1}\left\{\frac{e^{-2s}}{s^2+6s-7}\right\} = U(x-2)\, f(x-2)$

$\frac{1}{s^2+6s-7} = \frac{1}{(s+7)(s-1)} = \frac{A}{s+7} + \frac{B}{s-1}$

$s=-7 \quad s=1$

$A = -\frac{1}{8} \quad B = \frac{1}{8}$

$\ell^{-1}\{F(s)\} = f(x) = \ell^{-1}\left\{\frac{1}{s^2+6s-7}\right\} = \ell^{-1}\left\{\frac{-1}{8(s+7)} + \frac{1}{8(s-1)}\right\}$

$= -\frac{1}{8}e^{-7x} + \frac{1}{8}e^{x}$

$\ell^{-1}\left\{\frac{e^{-2s}}{s^2+6s-7}\right\} = U(x-2)\left(\frac{-1}{8}e^{-7(x-2)} + \frac{1}{8}e^{x-2}\right)$

(vi) Calculate $\int_0^\pi cos(2x)e^{-3x}\ dx$ [Hint : maybe you want to use the fact that Co(2x) has period equals to π and you know $\ell\{cos(2x)\}$].

$$\ell\{\cos(2x)\} = \frac{1}{1-e^{-\pi s}} \int_0^\pi e^{-sx} \cdot f(x)\,dx$$

$$= \frac{1}{1-e^{-\pi s}} \cdot \int_0^\pi e^{-sx} \cos 2x\,dx$$

$$\ell\{\cos 2x\} = \frac{s}{s^2+4}$$

$$\int_0^\pi e^{-sx} \cos 2x\,dx = \frac{s(1-e^{-\pi s})}{s^2+4}$$

as $s \to -3$: $\int_0^\pi \cos 2x\, e^{-3x}\,dx = \boxed{\frac{-3(1-e^{3\pi})}{13}}$

(vii) $\ell^{-1}\{\frac{s+3}{(s+7)^3}\}$

$$\ell^{-1}\left\{\frac{s+7-4}{(s+7)^3}\right\}$$

$$* \ \ell^{-1}\left\{\frac{s-4}{s^3}\right\} = \ell^{-1}\left\{\frac{1}{s^2}\right\} - \frac{4}{2}\ell\left\{\frac{1.2}{s^3}\right\}$$

$$= x - 2x^2$$

$$\ell^{-1}\left\{\frac{s+3}{(s+7)^3}\right\} = (x-2x^2)\,e^{-7x}$$

$\cos 2x * y(x)$

QUESTION 2. (10 points) Solve for $y(x)$: Given $y'(x) = sin(2x) + 8\int_0^x cos(2r)y(x-r)\ dr$, where $y(0) = 0$.

$$sY(s) - y(0) = \frac{2}{s^2+4} + 8\,\ell\{\cos 2x * y(x)\}$$

$$sY(s) = \frac{2}{s^2+4} + 8\left(\frac{s}{s^2+4}\right)(Y(s))$$

$$sY(s) - \frac{8s}{s^2+4}Y(s) = \frac{2}{s^2+4}$$

$$\frac{s^3-4s}{s^2+4}Y(s) = \frac{2}{s^2+4} \quad ; \quad Y(s) = \frac{2}{s^3-4s} = \frac{2}{s(s^2-4)}$$

$$* \ \frac{1}{s(s-2)(s+2)} = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s+2}$$

$s=0$, $s=2$, $s=-2$

$A = -\frac{1}{4}$, $B = \frac{1}{8}$, $C = \frac{1}{8}$

$$= \frac{-1}{4s} + \frac{1}{8(s-2)} + \frac{1}{8(s+2)}$$

$$y(x) = \ell^{-1}\{Y(s)\} = 2\,\ell^{-1}\left\{\frac{-1}{4s} + \frac{1}{8(s-2)} + \frac{1}{8(s+2)}\right\}$$

$$= \boxed{2\left(-\frac{1}{4} + \frac{1}{8}e^{2x} + \frac{1}{8}e^{-2x}\right)}$$

QUESTION 3. (10 points) Solve for $y(x)$: Given $y' + 6y = U(x-2)e^{3x}$; $y(0) = 0$

$g(x)$

$sY(s) - y(0) + 6Y(s) = e^{-2s} \cdot e^{6} \cdot \frac{1}{s-3}$

$Y(s)(s+6) = \frac{e^{-2s} \cdot e^{6}}{s-3}$

$Y(s) = \frac{e^{-2s} \cdot e^{6}}{(s-3)(s+6)}$

$y(x) = \mathcal{L}^{-1}\{Y(s)\} = f(x-2)\,U(x-2)$

$= e^{6}\left(\frac{1}{9}e^{3(x-2)} - \frac{1}{9}e^{-6(x-2)}\right) \cdot U(x-2)$

$\mathcal{L}\{e^{3(x+2)}\} = \mathcal{L}\{e^{3x} e^{6}\} = e^{6}\mathcal{L}\{e^{3x}\} = e^{6} \cdot \frac{1}{s-3}$

$f(x) = \mathcal{L}^{-1}\left\{\frac{1}{(s-3)(s+6)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{9(s-3)} + \frac{-1}{9(s+6)}\right\} = \frac{1}{9}e^{3x} - \frac{1}{9}e^{-6x}$

$\frac{1}{(s-3)(s+6)} = \frac{A}{s-3} + \frac{B}{s+6}$, $s=3$, $s=-$, $A = \frac{1}{9}$, $B=$

QUESTION 4. (12 points) Solve for $y(t)$ and $x(t)$: Given $y'(t) + 2x(t) = 0$ and $-2y(t) + x'(t) = 2$, where $x(0) = 0$, $y(0) = 0$.

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~~$y'(t) + 2x(t) = 0$~~

$2x(t) + y'(t) = 0$
$x'(t) - 2y(t) = 2$, Apply laplace ,

$2X(s) + sY(s) - y(0) = 0$

$sX(s) - x(0) - 2Y(s) = \frac{2}{s}$

$X(s) = \frac{\det\begin{bmatrix} 0 & s \\ \frac{2}{s} & -2 \end{bmatrix}}{\det\begin{bmatrix} 2 & s \\ s & -2 \end{bmatrix}} = \frac{0-2}{-4-s^2} = \frac{-2}{-4-s^2} = \frac{2}{s^2+4}$

$x(t) = \mathcal{L}^{-1}\{X(s)\} = \mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\} = \boxed{\sin 2t}$

substitute to find $y(t)$:

$x(t) = \sin 2t$
$x'(t) = 2\cos 2t$

$x'(t) - 2y(t) = 2$
$2\cos 2t - 2y(t) = 2$
$2y(t) = 2\cos 2t - 2$
$\boxed{y(t) = \cos 2t - 1}$

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MTH 205, Differential Equations , Exam Two

Ayman Badawi

QUESTION 1. (i) **(10 points)** Find the general solution to : $y^{(4)} + 4y^{(2)} = 0$

For y_h: $y = e^{\alpha x}$. $C(\alpha) = \alpha^2(\alpha^2 + 4) = 0$

$\alpha_1 = 0$, $\alpha_2 = 0$, $\alpha_3 = 2i$, $\alpha_4 = -2i$

$y_1 = 1$, $y_2 = 1x = x$, $y_3 = \cos(2x)$, $y_4 = \sin(2x)$

$y_g = y_h = c_1 + c_2 x + c_3 \cos(2x) + c_4 \sin(2x)$ ✓

(ii) **(10 points)** Find the general solution to : $y^{(4)} + 4y^{(2)} = 4x + 9$ [You may use (1) above]

$y_h = c_1 + c_2 x + c_3 \cos(2x) + c_4 \sin(2x)$

$y_p = (ax + b)x^2 = ax^3 + bx^2$

$y_p' = 3ax^2 + 2bx$

$y_p'' = 6ax + 2b$

$y_p^{(3)} = 6a$

$y_p^{(4)} = 0$

$0 + 4(6ax + 2b) = 4x + 9$

$24ax + 8b = 4x + 9$

$24ax = 4x$ $\quad 8b = 9$

$24a = 4$ $\quad \boxed{b = 9/8}$ ✓

$\boxed{a = 1/6}$

$y_g = c_1 + c_2 x + c_3 \cos(2x) + c_4 \sin(2x) + \frac{1}{6}x^3 + \frac{9}{8}x^2$

QUESTION 2. (15 points) Find the general solution to : $(x+1)y^{(2)} + y' = 1$

$y_1 = 1$
$y' = 0$
$y'' = 0$

To find y_h: use reduced 2nd order:

first, $(x+1)y^{(2)} + y' = 0$

$$y^{(2)} + \frac{1}{x+1} y' = 0$$

$y_1 = \text{constant} = 1$

$$y_2 = y_1 \int \frac{e^{-\int Q(x)}}{y_1^2} = \int e^{-\int \frac{1}{x+1} dx} = \int e^{-\ln(x+1)} = \int \frac{1}{x+1}$$

$$= \ln(x+1)$$

$$y_h = C_1 + C_2 \ln(x+1)$$

To find y_p: use variation

$y_p = f_1 + f_2 \ln(x+1)$

(2)

Setting: $f_1' y_1 + f_2' y_2 = 0$

$f_1' y_1' + f_2' y_2' = \frac{k(x)}{(x+1)}$

$\Rightarrow f_1' + f_2' \ln(x+1) = 0$

$0 + f_2' \left(\frac{1}{x+1}\right) = \frac{1}{x+1}$

$\hookrightarrow f_2' = 1$

$f_2 = \int 1 \, dx = x$

$f_1' + \ln(x+1) = 0$

$f_1' = -\ln(x+1)$

$f_1 = -\int \ln(x+1) \, dx$

$u = \ln(x+1)$ $dv = dx$
$du = \frac{1}{x+1} dx$ $v = x$

$$\int \frac{x}{x+1} dx = \int \frac{x+1}{x+1} dx + \int \frac{-1}{x+1} dx = x + \ln(x+1)$$

$f_1 = -x\ln(x+1) + \int \frac{x}{x+1}$

$u = x$ $dv = \frac{1}{x+1}$
$du = 1\,dx$ $V = \ln(x+1)$

$-\int \ln(x+1) dx = -x\ln(x+1) + x\ln(x+1) - \int \ln(x+1) dx$

$-2 f_1 = -2x\ln(x+1)$

$f_1 = x\ln(x+1)$

$-\int \ln(x+1) dx =$
$-x\ln(x+1) + x$
$-\ln(x+1)$
$-x + \ln(x+1)(-x$

$y_g = C_1 + C_2 \ln(x+1) + x\ln(x+1) + x\ln(x+1)$

$= C_1 + C_2 \ln(x+1) + 2x\ln(x+1)$

QUESTION 3. (10 points) Find the general solution to : $sin(x)y'+cos(x)y = sin(x)e^{cos(x)}$. Give me one particular solution to the D.E.

$$y' + \frac{\cos(x)}{\sin(x)} y = e^{\cos(x)}$$

$$\int \frac{\cos x}{\sin x} dx \quad u = \sin x \quad du = \cos x\, dx \quad dx = \frac{du}{\cos x}$$

$$= \int \frac{1}{u} du = \ln u = \ln \sin x$$

$$I = e^{\int \frac{\cos x}{\sin x} dx} = e^{\ln(\sin(x))} = \sin(x)$$

$$y = \frac{\int I \cdot f(x)}{I} = \frac{\int \sin x \; e^{\cos(x)}}{\sin x}$$

$$u = \cos x \quad du = -\sin x\, dx \quad dx = \frac{du}{-\sin x}$$

$$\int \sin x\, e^{\cos x} dx = -\int e^u du = -e^u$$

$$y = \frac{-e^{\cos x}}{\sin x} + \frac{c}{\sin x}$$

$$y_p = \frac{-e^{\cos x}}{\sin x}$$

QUESTION 4. (10 points) Find the general solution to : $(x-2)y' - y = \frac{x-2}{x+3}$

make coeff. of $y' = 1$

$$y' - \frac{1}{x-2} y = \frac{1}{x+3}$$

$$I = e^{\int \Theta(x)\, dx} = e^{\int \frac{-1}{x-2} dx} = e^{-\ln(x-2)} = \frac{1}{x-2}$$

$$y = \frac{\int I \cdot f(x)\, dx}{I} = \frac{\int \frac{1}{(x-2)} \cdot \frac{1}{(x+3)} dx}{\frac{1}{x-2}} = \frac{\int (x^2 + x - 6)^{-1} dx}{\frac{1}{x-2}}$$

$$= \frac{\int x^{-2} + x^{-1} - \frac{1}{6}\, dx}{\frac{1}{x-2}} = \frac{-x^{-1} + \ln x - \frac{1}{6}x + C}{\frac{1}{x-2}}$$

$$y_g = (x-2)\left(\frac{-1}{x} + \ln x - \frac{1}{6}x\right) + C(x-2)$$

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QUESTION 5. (10 points) Find the general solution to : $\sqrt[3]{x^2}y^{(2)} + \frac{3}{\sqrt[3]{x}}y' + \frac{5}{\sqrt[3]{x^4}}y = 0$

use $y = x^n$
$y' = n x^{n-1}$
$y'' = (n^2-n)x^{n-2}$

$x^{2/3}(n^2-n)x^{n-2} + 3x^{-1/3} n x^{n-1} + 5x^{-4/3}x^n = 0$

$x^{n-4/3}(n^2-n+3n+5) = 0$

$x^{n-4/3}(n^2+2n+5) = 0$

$n^2+2n+5 = 0 \rightarrow n_1 = -1+2i$, $n_2 = -1-2i$

$y = y_h = c_1 x^{-1}\cos(2\ln x) + c_2 x^{-1}\sin(2\ln x)$

QUESTION 6. (15 points) Find the general solution to : $y' + \frac{1}{x+1}y = (x+1)e^{ln(x+1)}y^3$

$n = 3$
$1-n = 1-3 = -2$

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$\boxed{U = y^{-2}}$ $U' - \frac{2}{x+1}U = -2(x+1)^2$

$I = e^{\int \frac{-2}{x+1}dx} = e^{-2\ln(x+1)} = \frac{1}{(x+1)^2}$

$U = \frac{\int I \cdot f(x)}{I} = \frac{\int \frac{1}{(x+1)^2}\cdot (x+1)^2(-2)\,dx}{\frac{1}{(x+1)^2}}$

$U = -2x(x+1)^2 + c(x+1)^2$

$y = U^{-1/2} = \sqrt{\frac{1}{-2x(x+1)^2 + c(x+1)^2}}$

MTH 205, Differential Equations , Exam Three

Ayman Badawi

QUESTION 1. (15 points)

(i) Find an equation for the charge $q(t)$ on the Capacitor of an LRC circuit, when $L = 0.5$, R = 20, and $C = 0.01$ and $E(t) = 150$. Given $q(0) = 1$ and $i(0) = 0$.

(ii) What will be the current after 1 minute, i.e. find $i(1)$.

QUESTION 2. (12 points) You left your room this morning and you made sure that the AC is turned off. Later on in the day, you returned to your room and you discovered that the room Temperature is $100F$. So quickly you turned on the AC and you fixed the AC temperature at $70F$. After 5 minutes, the room Temperature became $90F$.

a) How long will it take before the ROOM Temperature reaches 75F?

b) Will the room temperature ever reaches $70F$? Mathematically Explain (Why yes or Why No)

QUESTION 3. (12 points) A large tank is partially filled with 100 gallons of water in which 10 pounds of salt is dissolved. A mixture containing 0.5 pound of salt per gallon is pumped into the tank at rate of 6 gal/min. The well-solution is then pumped out at rate 4 gal/min.

a) What will be the concentration of the salt in the tank after 30 minutes?

b) If an overflow occurred after 200 minutes, what is the capacity of the tank?

QUESTION 4. (16 points) a) Solve the D.E: $\dfrac{dy}{dx} = \dfrac{\sqrt{x+y+1}}{(\sqrt{x+y+1} \ + \ 4)^5} \ - \ 1$

b) Solve the D.E : $\dfrac{dy}{dx} = \dfrac{e^{4x+y}}{(3x+y)^2 \ +1} \ - 3$

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MTH 205, Differential Equations , Final Exam

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QUESTION 1. (10 points) Find the general solution to $y^{(2)} + \frac{y'}{x} - \frac{y}{x^2} = \frac{2}{x(x+1)}$

QUESTION 2. (10 points) Find the general solution to $y^{(2)} - \frac{2y'}{2x+1} = (2x+1)^2$

QUESTION 3. (10 points) Find the general solution to $y^{(5)} - 4y^{(4)} + 8y^{(3)} = 2$

QUESTION 4. (10 points) Find $y(x)$ such that $y' = x + \int_0^x \; y(r)\; dr, y(0) = 0$

QUESTION 5. (10 points)

a) Find $\ell^{-1}\{\frac{e^{-s}}{(s+1)^2+1}\}$

b) Find $\ell^{-1}\{\frac{s+2}{(s-3)^3}\}$

QUESTION 6. (10 points) Find the general solution to $\dfrac{dy}{dx} = \dfrac{cos(x)e^{(sin(x)+2+x+y)}}{1+x+y} - 1$

QUESTION 7. (10 points) Find the general solution to $\dfrac{dy}{dx} = \dfrac{x+e^x-sin(y)-2yx+4}{y+e^y+xcos(y)+x^2+2}$

QUESTION 8. (10 points) Solve for $x(t)$, $y(t)$ where
$x(t) - y'(t) = 0$
$x'(t) - y'(t) = 0, x(0) = 3, y(0) = 1$

QUESTION 9. (10 points) Find the general solution to $y' + \frac{y}{x} = y^{\frac{3}{2}}(1 + \sqrt{x})^5$

QUESTION 10. (10 points) a) An object weighing 8 pounds stretches a spring 2feet. Assume that a force numerically equals to 2 times the velocity of the motion $x(t)$ acts on the system. Determine the equation of motion $x(t)$ if the object is initially released from the equilibrium position with an upward velocity $3ft/s$.

b) Will the spring ever return to the equilibrium position? explain

c) If the answer to (b)is no, then at any time $t > 0$, will the motion of the spring be above or below the equilibrium position? Explain

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